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THE METHOD OF LUMINOUS INTENSITY SPATIAL DISTRIBUTION CALCULATION FOR LIGHT SOURCES BASED ON LEDs WITH DIFFERENT OPTICAL AXIS ORIENTATION

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There exist design projects of light fixtures with light emitting diodes (LEDs), in which LED radiators (or LED modules-LEDM) could be utilized with light distribution patterns partially or fully different in various viewing directions from each of the LEDs in the fixture (including LEDs with secondary optics). However, commercially available LEDMs of this type with required characteristics and of good quality actually are unavailable. Fundamental approaches to designing methods for development of such devices, are also lacking, particularly for calculations of LEDMs with a required luminous intensity spatial distribution (LISD).

Therefore, it seems meaningful to develop a method of calculation of a LISD for LEDMs, including asymmetrical and partially or fully multidirectional LEDs of one or several types (with specific "typical" LISD).

We assumed that each of the LEDMs, independent of their design, has a photometrically equivalent LEDM in the form of a set of multidirectional plates

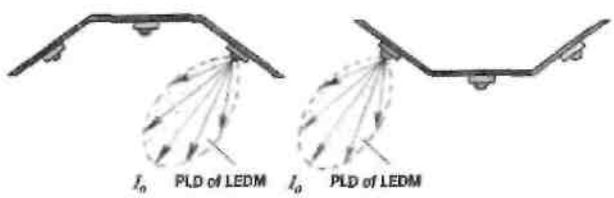

Fig. 1. An example of photometric equivalency of convex (a) and concave (b) forms of photometric body (PhB) of LEDM (see explanations in text)

with mounted on them LEDs of one or several types; the axes of all LEDs are perpendicular to the corresponding plates.

Our method is based on the substitution of visual models of corresponding photometrically equivalent LEDMs for actual designs of LEDMs (in some cases they may coincide). In doing so, the photometrically equivalent LEDMs can be either convex (if their radiating side is an outside one), concave (if their radiating side is an inside one), or convex-concave ones.

Moreover, we used the following limitations: 1) the LISD of each LED is axisymmetrical relative to the LED's optical axis; 2) the LEDs of the same type have similar LISD; 3) the LEDs do not shield each other photometrically and are not shielded by other LED construction elements; 4) the LEDs area point light sources.

Note that: a) the limitation 3 combined with other simplifications, reduces all three above mentioned shapes of the photometrically equivalent LEDM to just one either convex or concave (Fig. 1); b) the limitation 4, in particular, makes parallel planes going through the photometrically equivalent LEDM photometrically interchangeable; as well as, for example, various systems of right-angled coordinates with corresponding parallel axes and centers located in the overall dimensions of the photometrically equivalent LEDM.

The above limitations are taken into account in the optics-geometrical diagram of the calculations shown in Fig. 2-5.

Fig. 2 shows a concave photometrically equivalent LEDM, every plate of which has different numbers of various type LEDs (a), and an i-plate in a system of right-angled coordinates *(X, Y, Z)*. There, $\boldsymbol{I_0}$ is the vector of axial luminous intensity of a single j-type LED on a given i-plate. It is perpendicular to the plane of radiating side of the plate. This plane goes through the light centers of all types of LEDs (due to limitation 4), located on the given plate, cutting off the coordinate axes intercepts a, b, and c. The plane can be described by the equation [1] as follows:

$$x/a + y/b + z/c = 1 \qquad (1)$$

Fig. 2 also shows a local system of Cartesian coordinates *(X', Y', Z')*, obtained by the parallel transfer of the coordinate system *(X, Y, Z)* to the location with the center in the point ***O'***. This point is a light center of the corresponding LED, and from this point the vector $\boldsymbol{I_0}$ is plotted. Also, for comparison, the coordinate system *(X", Y", Z")* for photometrically equivalent LEDM as a whole it shows.

Fig. 3 shows several additional geometrical definitions: Θ_{xz}, Θ_{xz} and Θ_{φ} are the vector's $\boldsymbol{I_0}$ angular projections to the coordinate planes *xz, yz,* and an arbitrary azimuthal semi-plane going at an angle φ to one of the coordinate plane *(xz)* (further in the paper as j-semi-plane); γ_{xz}, γ_{yz} and γ_{φ} are the angles between the coordinate axis OZ and the rectangular projections of vector $\boldsymbol{I_0}$ respectively on the coordinate planes *xz* and *yz* and the (φ-semi-plane (and parallel to them planes).

Each pair of angles Θ and γ with corresponding indexes *(xz, yz or j)* determines space-angle position of the given plate and the corresponding vector $\boldsymbol{I_0}$ (i.e. orientation of the given LED) in the above-mentioned coordinate systems: *(X, Y, Z), (X', Y, Z')* and *(X", Y", Z")*.

These angles *(Θ_{xz} γ_{xz} or Θ_{yz} γ_{yz})* enter as input parameters for each plate (and the LED located on it):

- either directly (e.g. when developing a new LEDM),
- or indirectly if the equation for radiating side of the plate has been found before, (it can be easily done, for example, upon estimating LISD of existing LEDM or their combinations).

In the latter case the above-mentioned equation determines the corresponding intercepts *a, b,* and *c*

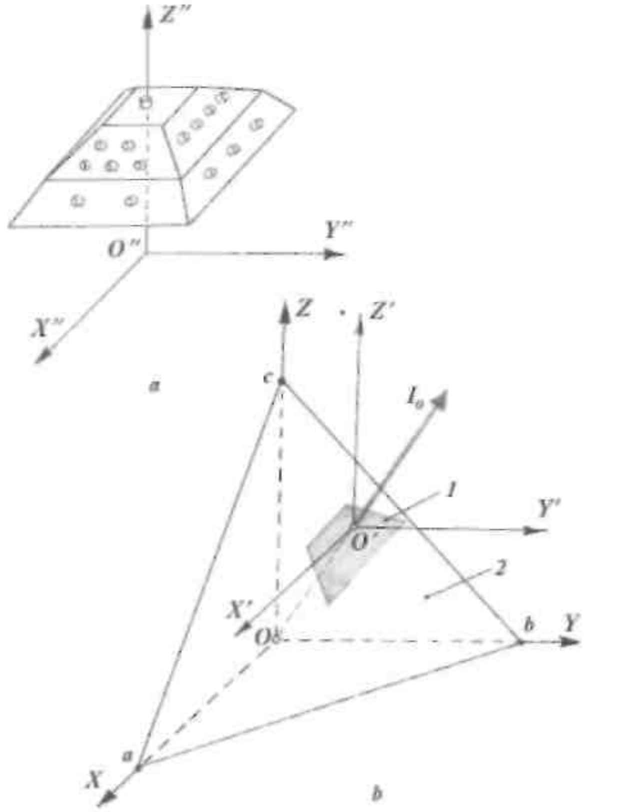

Fig. 2. Optics-geometrical diagram of the calculations (see explanations in text):

1 is i-plate PhE of LEDM; *2* is LED plane of i-side of PhE of LEDM

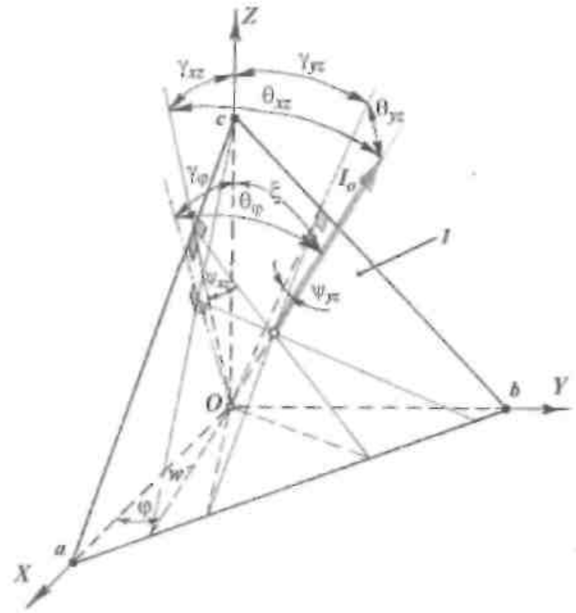

Fig. 3. Optics-geometrical diagram of the calculations (see explanations in text):

1 is a plane of radiating side of plate i of PhE LEDM (the plate is not shown)

(Fig. 2), and by using them also determines the above mentioned angles:

$$\Theta_{xz} = \pi/2 - \psi_{xz}$$

$$\psi_{xz} = \arccos[1 + (b/c)^2 + (b/a)^2]^{-1/2}$$

$$\Theta_{yz} = \pi/2 - \psi_{yz}$$

$$\psi_{yz} = \arccos[1 + (a/b)^2 + (a/c)^2]^{-1/2}$$

As can be seen in Fig. 3, these equations follow from the equation for the plane Eq. (1), and a well-known formula [2] for the angles between two planes (here, between the radiating side of the plate and the coordinate planes *xz* and *yz* respectively);

For the γ-angles the equations are:

$$tn\gamma_{xz} = c/a \quad and \quad tn\gamma_{yz} = c/b \qquad (2)$$

(From Fig. 3 one can also see that in right triangles with legs *a* and *c*, and *b* and *c*, the leg c has the alternate angles γ_{xz} and γ_{yz})-

Also, for the mentioned angles, the following relation is valid:

$$\cos\Theta_{xz}\cos\gamma_{xz} = \cos\Theta_{yz}\cos\gamma_{yz} = $$
$$= \cos\Theta_{\varphi}\cos\gamma_{\varphi} (= \cos\xi)$$

which follows from Fig. 3 and elementary trigonometry [2, 3] (the three right dihedral angles lie opposite the flat angle ξ).

Fig. 4 shows, in the above-mentioned system of coordinates *(X', Y', Z'),* luminous intensity distribution (LID) of a single LED of j-type located on the plate *i*, and the cross-sections of this LISD by the coordinate (and simultaneously azimuthal) planes *x'z'* and *y'z'*- The contours of these cross-sections are but the corresponding luminous intensity distributions (LIDs) of the specific LED on the given plate in the shown planes. Here the role of polar angles is played by the angles α_{xz} and α_{yz}, which correspond to the angles β_{xz} and β_{yz};, directly connecting the above-mentioned LIDs (to be found) and LISD LED (known). The latter are determined by the following equations that are analogous to Eq. (3):

$$\cos\beta_{xz} = \cos\Theta_{xz}\cos(\alpha_{xz} - \gamma_{xz}), \qquad (4)$$

$$\cos\beta_{yz} = \cos\Theta_{yz}\cos(\alpha_{yz} - \gamma_{yz}) \qquad (5)$$

Here the corresponding right dihedral angles are alternate to the flat angles β_{xz}, β_{yz} and β_{φ} Due

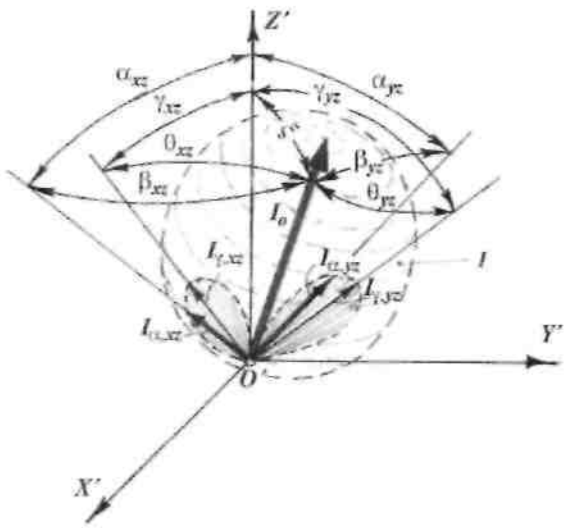

Fig. 4. Optics-geometrical diagram of the calculations (see explanations in text):

1 is LID of a single LED of j type on i plate of LISD LEDM (the plate is not shown)

to evenness of the functions $\cos(\alpha_{xz} - \gamma_{xz})$ and $\cos(\alpha_{yz} - \gamma_{yz})$their arguments' sign does not affect results.

From Eq. (3-5) the angles β_{xz} and β_{yz} can be found as functions of the corresponding polar angles α_{xz} and α_{yz}, as well as LID LED in the coordinate planes *xz*(φ = 0°) and *yz* (φ = 90°). The latter. in some cases (for example, when there is LEDM symmetry with respect to both coordinate plates) mostly determines the photometrically equivalent LEDM.

For a general case, Fig. 5 shows a cross-section of LISD LED of Fig. 4 by a φ-semi-plane. The contour of this cross-section is an LID curve of a single LED of the φ-type on the plate i on the given semi-plane. Then the β_{φ} angles that directly determine an LID (to be found using the known LISD LED) cone-spond to the polar angles α_{φ} in this semi-plane.

Thus, the gist of our calculations is finding angles β_{φ}. Note, that from the expression $\cos\beta_{\varphi} = \cos\Theta_{\varphi}\cos(\alpha_{\varphi} - \gamma_{\varphi})$ that is similar to Eqs. (4 and 5) and uses Eq. (3), it follows:

$$\beta_{\varphi} = \arccos[\cos\Theta_{xz}\cos\gamma_{yz}\cos(\alpha_{\varphi} - \gamma_{\varphi})/\cos\gamma_{\varphi}] =$$
$$= \arccos[\cos\Theta_{yz}\cos\gamma_{yz}\cos(\alpha_{\varphi} - \gamma_{\varphi})/\cos\gamma_{\varphi}] \quad (6)$$

And now in order to arrive at an equation for β_{φ} to be used in calculations, one should express the angle γ_{φ}. through the input quantities (in other words to

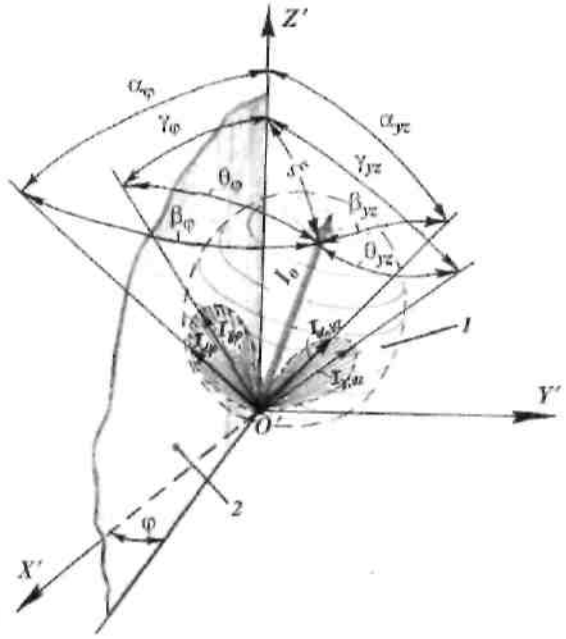

Fig. 5. Optics-geometrical diagram of the calculations (see explanations in text):

1 is LID from Fig.2; 2 is φ-semi-plane

eliminate γ_φ). For this purpose let us consider the straight triangle in Fig. 3 formed by the coordinate axis $0X$, $0Y$, and the intercept line of the xy plane and the plane, in which the radiating plate is located, as well as the segment of length w in this triangle, formed by the line of interception of φ-semi-plane and the xy plane. Then one writes:

$$0.5 \cdot a \cdot w \sin\varphi + 0.5 \cdot b \cdot w \cdot \sin(90^\circ - j) = 0.5 \cdot a \cdot b$$

From which it follows:

$$w = a \cdot b / (a \cdot \sin\varphi + b \cdot \cos j). \quad (7)$$

One can see that in the straight triangle with legs w and c, the c-leg has γ_φ as an alternate angle found from:

$$tn\gamma_\varphi = c / w \quad (8)$$

And, Finally, from Eq. (2), (7), and (8) it follows:

$$\gamma_\varphi = arctn(tn\gamma_{xz} \cos\varphi + tn\gamma_{yz} \sin\varphi).$$

This equation can be used together with Eqs. (4-6) for calculating angles β_φ for each plate.

The LED light intensities for all polar angles α_φ are found making use of angles β_φ for the i-plate and the known LISD parameters of a single LED located

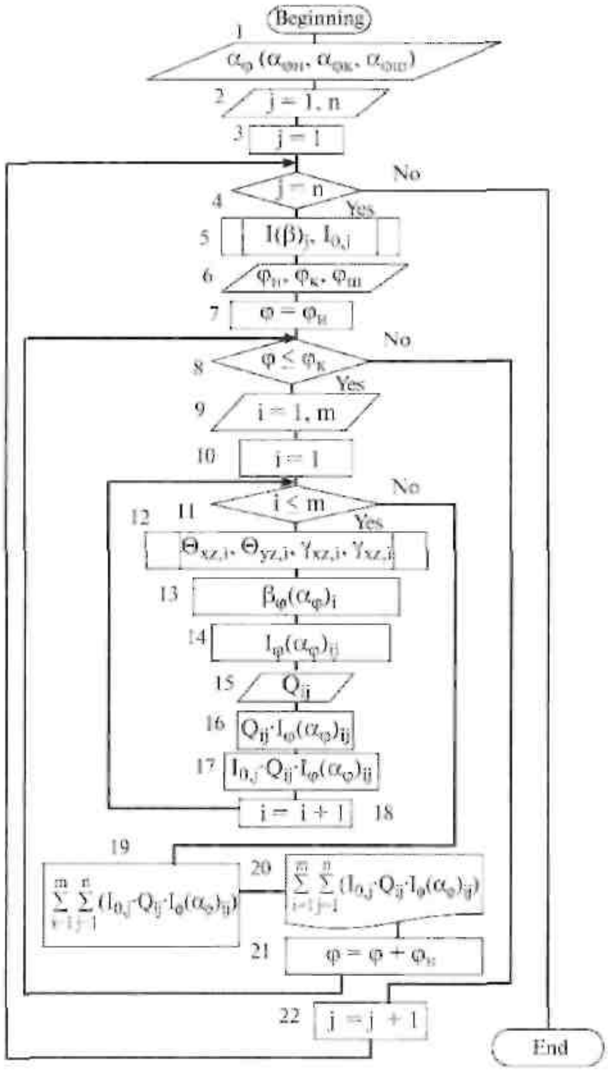

Fig. 6. Diagram of algorithm of calculation of LISD of LEDM with partial or complete multidirectional orientation of LED (a version):

1 is input of a set of polar angles (from $\alpha_{\varphi H}$ to $\alpha_{\varphi K}$, with step $\alpha_{\varphi III}$); 2, 3, and 4 are inputs of a list of LED types in LEDM, their starting point and verification: 5 is input of approximation parameters of LI D and absolute value of axial light intensity of a single LED of j-type (if necessary corresponding subprograms can be used (see text); 6. 7, and 8 are input of starting and final azimuthal angles φ and step of their variation, their starting point and verification; 9,10, and 11 are input of a list of plate numbers, its starting point and verification; 12 is input of coordinate angles of i-plate (if necessary corresponding subprograms can be used (see text); *13* is calculation of angles βφ for *i*-plate; 14 is calculation of relative LIDs of a single LED of *j*-type on *i*-plate in φ semi-plane (i.e. at azimuthal angle φ); 15 is input of number of *j*-type LEDs on i-plate; 16 is calculation of relative LID of a set of j-type LEDs on *i*-plate in φ semi-plane; 17 is absolutization of results of previous step; 18 is variation of plate number; 19 is calculation of LID LEDM in φ semi-plane; 20 is sequential output of LID LEDM for all azimuthal φ angles (at φ semi-planes) and their tabulation; 21 is variation of azimuthal angle φ; 22 is variation of number of LEDM type

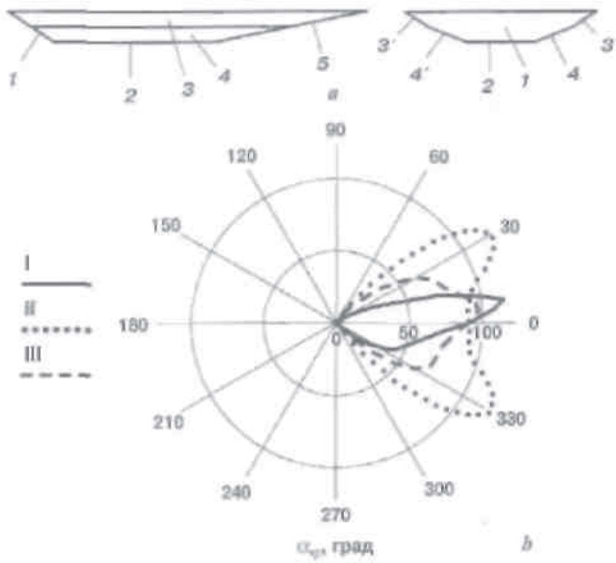

Fig. 7. Convex LISD LEDM with one (longitudinal) symmetry plane.

(a) numbers correspond to plates considered and their parameters: (/ - angle of plate to horizontal axis = 22°/ angle, at which light intensity is half of maximum = 32° / axial light intensity = 160 cd / number of such LEDs on this plate =3); *2*- (0° / 32° / 160 cd / 1); 3 and 3' - (30° /32° / 170 cd / 8); 4 and 4' - (10° /32° /48.5 cd / 1); 5 - (8° /32° /160 cd / 3) and (8°/ 12°/ 570 cd / 1);

(b) results of calculation of LID in cd/1000 lm in three azimuthal planes (six corresponding semi-planes). I is LID in main longitudinal plane (90°-270°); II is in main transverse plane 0° - 180°); III is LID in plane (150°-330°)

on the plate. The set of LIDs of a single LED of the *j*-type on the *i*-plate is taken into account consecutively one by one for all azimuthal angles (φ, i.e. we obtained a set of LIDs of a single LED of *j*-type on i-plate in various (φ semi-planes.

In our next step with the goal of finding LISD LEDM it is necessary to multiply the obtained result by the number of LEDs of *j*-type on the *i*-plate and (if required) by the absolute value factor (which is the absolute value of axial light intensity). The above procedure should be repeated for all the other plates and LED types used in LEDM. The obtained "partial" LIDs should be summed up in the given φ - semi-plates, thus arriving at the set of LIDs for the whole LEDM in these azimuthal semi-planes, i.e. the LISD of LEDM we have been seeking.

Our mathematical model is comprised of three cycles (Fig. 6). The internal cycle varies the number of plates (*i*); the middle one — values of azimuthal angle φ, and the outer one — the LED type (*j*). Actually, the algorithm tabulates LID in various LEDM LIDs in various azimuthal semi-planes, which just determines LISD of LEDM.

Due to it's analytical character, the method is inherently accurate, simple, and calculationally transparent (each step of calculations can be checked easily with a calculator for engineering estimates).

In particular:

- Computer realization of the method does not cause problems for lighting engineers familiar with MathCad, MatLab, or other popular software;
- The method can also be used with more sophisticated software.

An example of realization of this method using MathCad is presented in Fig. 7.

There is LISD partially represented by a set of LIDs in six azimuthal semi-planes or three corresponding azimuthal planes. The calculation m a d e use of LEDs of several types (with different LISDs) placed differently on the plate. (LISDs of LED were approximated by an internal function *Genfit).*

The method has been successfully used by the authors for both developing new LEDMs with various LISDDs and for evaluation of the commercial LEDMs or their combination.

References

1. *Bronshtein, I.N., Semendyaev, K.A.* Handbook on mathematics. M. Nauka. 1965.

2. Pogorelov, A.V. Geometry. M. Nauka. 1993.

3. *Ivlev, B.* Dihedral and trihedral angles. http://kvant.mccme.ru

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